

Accelerated Failure Time Models

Recall

For several common distributions for T , we can write

$$Y = \log T = \mu_Y + \sigma W,$$

hinting at a **regression model** of sorts.

Introducing Covariates

Suppose we are interested in the impact of X on Y . It is natural to take

$$E[Y_i|X_i] = \eta_i = X_i'\beta,$$

which renders

$$Y_i = \eta_i + \sigma W.$$

Doing this produces an **accelerated failure time model**.

Formally...

For a random time-to-event outcome, T , an **accelerated failure time** (AFT) model proposes that

$$Y_i = \log T_i = x_i' \beta + W_i = \eta_i + W_i,$$

where $W_i \stackrel{iid}{\sim} \Omega$ for some mean-zero error distribution.

The specific form of AFT is specified by selecting the distribution for W .

Properties of AFTs

Why the Name?

Notice that, under an AFT,

$$T_i = \exp(\eta_i) T_{0i},$$

where $T_{0i} = \exp(W_i)$.

This represents a **multiplicative rescaling** of the expected time.

Relevant Quantities

If we consider the **original time scale**, we can show that

$$f(t) = f_0(e^{-\eta}t)e^{-\eta}$$

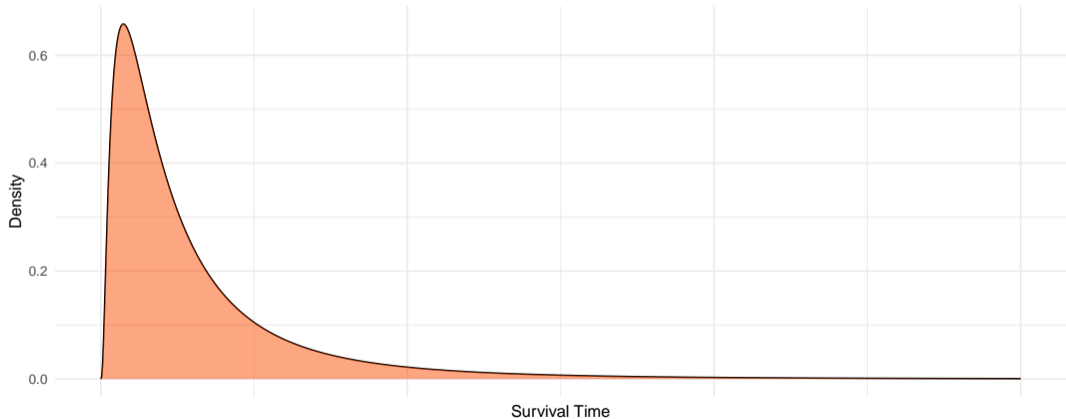
$$S(t) = S_0(e^{-\eta}t)$$

$$h(t) = h_0(e^{-\eta}t)e^{-\eta}.$$

Example: Log-Normal

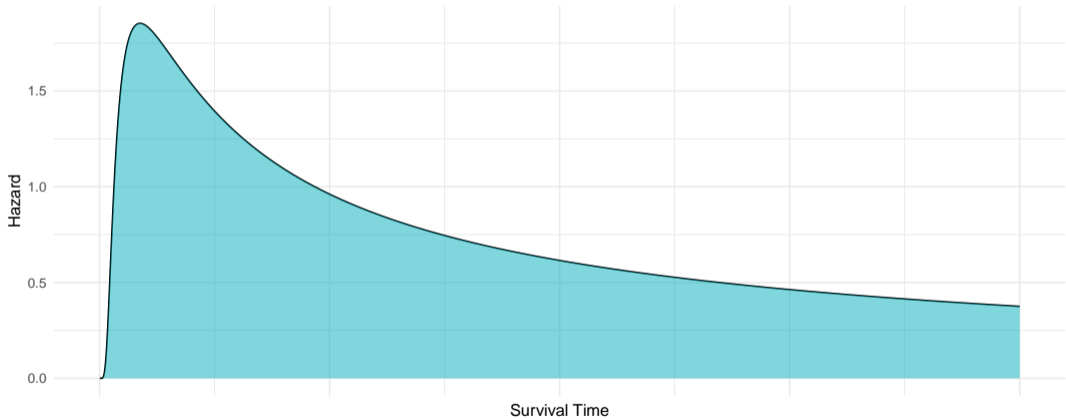
Standard Regression Quantities

In a **standard linear regression**, we would take $W \sim N(0, \sigma^2)$. Doing so implies a **log-normal** distribution for T . On the surface, this seems *okay*.



The Hazard Function

But, it produces a somewhat **unhelpful** hazard function.



List of Common Distributions

Distribution of T	Distribution of W
Exponential(ρ)	Extreme Value
Weibull(κ, ρ)	Extreme Value
Log-Logistic(κ, ρ)	Logistic
Log-Normal(μ, σ)	Normal

Summary

- ▶ Accelerated failure time (AFT) models are **regression models** for location-scale families.

Summary

- ▶ Accelerated failure time (AFT) models are **regression models** for location-scale families.
- ▶ They represent a **multiplicative rescaling** of the baseline time-to-event.

Summary

- ▶ Accelerated failure time (AFT) models are **regression models** for location-scale families.
- ▶ They represent a **multiplicative rescaling** of the baseline time-to-event.
- ▶ Can be interpreted as standard regression on the **log scale**.

Summary

- ▶ Accelerated failure time (AFT) models are **regression models** for location-scale families.
- ▶ They represent a **multiplicative rescaling** of the baseline time-to-event.
- ▶ Can be interpreted as standard regression on the **log scale**.
- ▶ They are **parametrically selected** based on the corresponding error distribution.