Approaches to Analysis So Far

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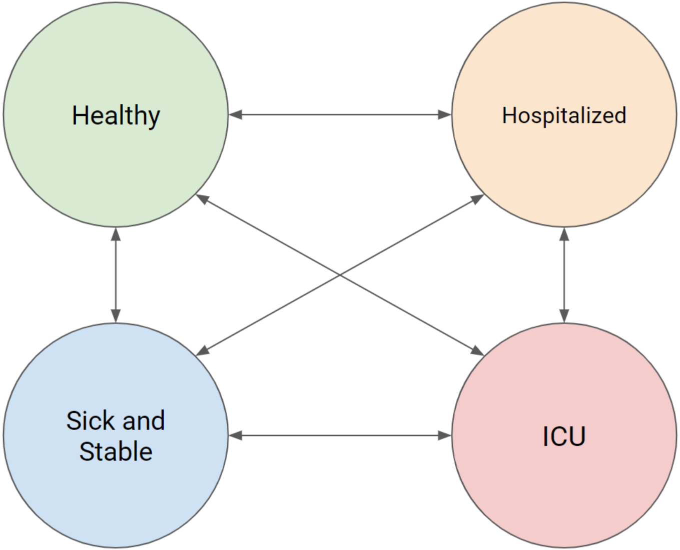
How might we be able to handle **categorical** data?

Example . . .



Do You Remember Stochastic Processes?

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Can maybe exploit **Markov chains**?

Markov Property

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A first-order Markov chain is thus characterized **entirely** by the transition probabilities,

$$p_{\ell m}(t) = P(Y_t = m | \mathcal{H}_t) = P(Y_t = m | Y_{t-1} = \ell),$$

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If the Markov chain is **time homogenous** then

$$p_{\ell m}(t) = p_{\ell m}(t') = p_{\ell m},$$

for all $t \neq t'$.

Transition Models for Longitudinal Data

First-Order Transition Model

If we ignore covariates, and consider a **first order** Markov model, with equally spaced observations, then the parameters of interest will be

$$p_{\ell m}(t_j) = P(Y_{ij} = m | Y_{i,j-1} = \ell).$$

The likelihood will take the form of

$$L(\mathbf{p}) = \prod_{i=1}^{n} P(Y_{il}) \prod_{j=2}^{k} P(Y_{ij} | Y_{i,j-1}) = \prod_{i=1}^{n} \pi_{Y_{i1}} \prod_{j=2}^{k} p_{Y_{i,j-1}, Y_{i,j}}(t_j).$$

Likelihood Estimators

Maximizing the likelihood function, results in estimators given by

$$\widehat{p}_{\ell,m}(t) = \frac{\{\#\ \ell \rightarrow m \text{ transitions at } t_j\}}{\{\# \text{ of subjects in } \ell \text{ at } t_{j-1}\}}.$$

If we make the **time homogenous** assumption we get

$$\widehat{p}_{\ell,m} = \frac{\sum_{j=2}^{k}\{\#\ \ell \rightarrow m \text{ transitions at } t_j\}}{\sum_{j=2}^{k}\{\# \text{ of subjects in } \ell \text{ at } t_{j-1}\}}.$$

But . . . variates?

Logistic Transition Models

Consider the **first-order time-homogenous** model, with **binary data**.

Define $\mu_{ij}^{C} = E[Y_{ij}|Y_{i,j-1}] = P(Y_{ij} = 1|Y_{i,j-1})$ We can model this as

$$\text{logit}(\mu_{ij}^{C}) = \alpha_0 + \alpha_1 y_{i,j-1}.$$

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This is easily extended to a **second-order time-homogenous model** by simply taking

$$\text{logit}(\mu_{ij}^C) = \alpha_0 + \alpha_1 y_{i,j-1} + \alpha_2 y_{i,j-2}.$$

Logistic Transition Models, with Additional Covariates

If we are also interested in the impact of x_{ij} then consider that

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allows for ...

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1. Transition probabilities to depend on whatever measured factors.
2. Transition probabilities to differ between $Y_{i,j-1} = 0$ (defined by β) and $Y_{i,j-1} = 1$ (defined by $\beta + \alpha$).
3. Can be readily expanded to second-order (or higher) using additional terms!

Estimating in Practice

We can write down the likelihood, under the assumption of an r^{th} order Markov chain. It requires **conditional likelihood**.

Using this approach **standard logistic regression** can be applied, with the correct lagged terms!

This treats the first r observations (for each individual) as fixed.

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- Can pose a longitudinal process as a **stochastic process** with an appropriate **Markovian** assumption.
- Can use standard likelihood theory to characterize the **transition probabilities**.
- Using (e.g.) **logistic regression**, variates can be accommodated for the estimation procedure.