

From the Population to the Individual: Mixed Effects Models

Consider. . .

NEWS RELEASE

Ontario Moving Three Regions to New Levels in the COVID-19 Response Framework

Province also adjusting capacity limits for some events in Grey-Lockdown

March 12, 2021

Variation in Longitudinal Data

Longitudinal analysis is primarily concerned with breaking down **within-** and **between-** subject variation.

Where does Variation Come From?

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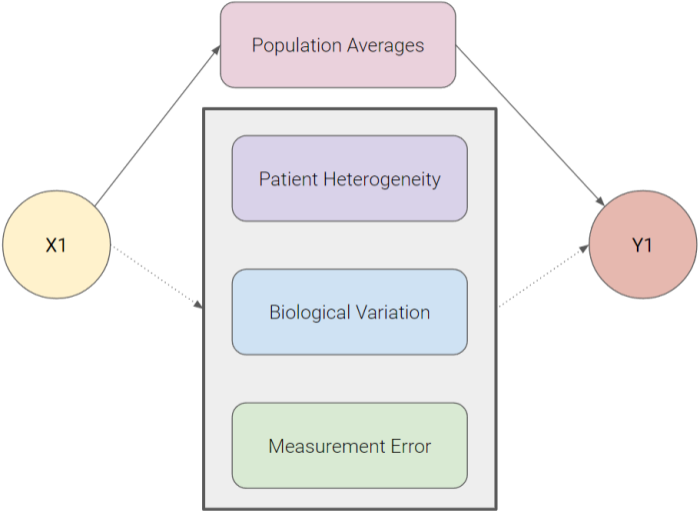
- ▶ Between patient heterogeneity.
- ▶ Within-individual biological variation.
- ▶ Measurement error.

How is Variation Accounted for in Marginal Models

In marginal models we make **no** explicit assumption regarding the sources of the variation.

We model the (population-level) **means** and assume variability within each individual and between the individuals in our sample.

How is Variation Accounted for in Marginal Models



Shortcomings of this Method

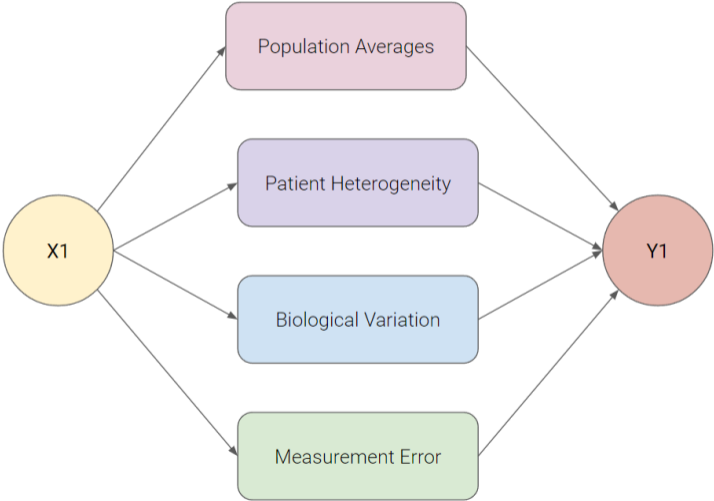
What if we care about making individual-level conclusions?

Introducing: Mixed Effects Models

Mixed Effects Models

The idea will be to specifically account for the within-person variability by assuming that outcomes are dictated by population-level and individual-level effects.

Mixed Effects Models



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- ▶ Population level effects

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- ▶ Patient-level heterogeneity

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- ▶ Individual measurement variation

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- ▶ We refer to the population-level effects as **fixed effects**.
- ▶ We refer to the individual-level effects as **random effects**.
- ▶ When we use both **fixed effects** and **random effects**, this is a **mixed effects model**.

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- ▶ Marginal models describe **population-averaged** effects without accounting for specific sources of variation.
- ▶ This means that individual-level effects **cannot** be estimated using marginal models.
- ▶ A mixed effects model specifies **fixed** and **random** effects to account for both population-level and individual-level effects, simultaneously.

Up Next. . .

We will learn how to **specify** and **estimate** mixed effects models.